Roll No. ☐☐☐☐☐☐☐☐☐☐☐☐☐   Total No. of Pages : 03

Total No. of Questions : 09

B.Tech. (2007-2010 Batches) (Sem.–1)

ENGINEERING MATHEMATICS-I

Subject Code : AM-101

Paper ID : [A0111]

Time : 3 Hrs.   Max. Marks : 60

INSTRUCTION TO CANDIDATES :

1. SECTION-A is COMPULSORY consisting of TEN questions carrying TWO marks each.
2. SECTION - B & C. have FOUR questions each.
3. Attempt any FIVE questions from SECTION B & C carrying EIGHT marks each.
4. Select atleast TWO questions from SECTION - B & C.

SECTION-A

1. **Write briefly :**

   (i) Sketch the graph of $y = |x| + 2x$.

   (ii) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

   (iii) If $u = \sin\left(\dfrac{x}{y}\right)$, $x = e^t$, $y = t^2$, find $\dfrac{du}{dt}$

   (iv) Find the percentage error in the area, of an ellipse when an error of +1 percent is made in measuring the major and minor axes.

   (v) Find the equation of the sphere whose center is $(2, -3, 4)$ and radius 5.

   (vi) Find the equation of the quadratic cylinder whose generators intersect the curve $ax^2 + by^2 = 2z, \, lx + my + nz = p$ and are parallel to z-axis.

   (vii) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \left[\int_0^{a\cos\theta} r\sqrt{a^2 - r^2} \; dr \right] d\theta$

(viii) Calculate $\iint_R f(x,y)dA$ for $f(x,y)=1-6x^2y$ and $R:0\le x\le 2,-1\le y\le 1$.

(ix) Discuss the convergence or divergence of the series $\sum \dfrac{\sec^{-1}n}{n^{1.3}}$.

(x) If sin $(A+iB)=x+iy$, prove that $\dfrac{x^2}{\cosh^2 B}+\dfrac{y^2}{\sinh^2 B}=1$.

SECTION - B

2. Graph the functions (a) $y=x^{\frac{5}{3}}-5x^{\frac{2}{3}}$ (b) $y=\left|x^2-1\right|$.

3. The region bounded by the curve $y=x^2+1$ and the line $y=-x+3$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

4. (i) If $z=xf\left(\dfrac{y}{x}\right)+g\left(\dfrac{y}{x}\right)$, show that $x^2\dfrac{\partial^2 z}{\partial x^2}+2xy\dfrac{\partial^2 z}{\partial x\partial y}+y^2\dfrac{\partial^2 z}{\partial y^2}=0$.

(ii) Verify Euler's theorem for the function, $u=\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)\left(x^n+y^n\right)$.

5. Find the minimum value of $x^2+y^2+z^2$, given that $ax+by+cz=p$.

SECTION-C

6. Find the equation of the sphere which passes through the points $(1,-4,3)$, $(1,-5,2)$, $(1,-3,0)$ and whose center lies on the plane, $x+y+z=0$.

7. Change the order of integration in the following integral and evaluate

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx.$$

8. Find the radius and interval of convergence for the following power series

$$\sum_{n=1}^{\infty} \frac{(4x-5)^{2n+1}}{n^{\frac{3}{2}}}$$

For what values of x, does the series converges (a) absolutely, (b) conditionally

9. If $C = \cos^2\theta - \frac{1}{3}\cos^3\theta\cos 3\theta + \frac{1}{5}\cos^5\theta\cos 5\theta - ...$, then prove that

tan $2C$ = 2 cot^2 θ.